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Optimized Image Compression Using Sparse Representations and Fourier Transform

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Abstract

Compression is an important step in saving and transmitting images. Compression serves mainly to remove the redundancy of the initial records. In this study, sparsity is used to compress images. Sparsity is an important subject so that during any process we can save a lot of computation. Since certain signals only use a few nonzero coefficients on an acceptable basis. Sparsity refers to the idea that a continuous-time signal can reflect a much lower rate of information than its bandwidth implies. Many physical signals or images are expressed in compressible form using the right basis function. Compression is relatively simple to implement on images, starting with high resolution. After that a transformed base is used in this project, and Fast Fourier transform is used. Then by using sparsity we can truncate and throw away a vast majority of those entries, which means we keep little Fourier coefficients that contribute the most important parts of the original image then after applying inverse Fourier transform we may get a high fidelity representation of the original image. In this case, my picture is compressed by using the sparsity method and to get almost the same efficiency and level of clearness almost 70% of the original size has almost saved.

Keywords: Image Compression, Sparsity, Compressed Sensing, Fast Fourier Transform.

1. Introduction

The implementation of data compression on digital images is the compression of images. The goal is, in essence, to minimize the redundancy of image data so that data can be processed or distributed optimally. Uncompressed multimedia data (graphics, audio, and video) requires significant bandwidth and storage space. With substantial improvements in the density of mass storage, processing speeds, and efficiency of digital communication networks, demand for data storage space, and bandwidth for data transmission continues to surpass the capacities of available technologies. In addition to the need for efficient means to encrypt signals and images, the recent rise in data-intensive multimedia web applications has made the compression of signals fundamental in storage technology and communication technologies. [1].

For most images, a key pattern is that the pixels neighboring are connected and therefore contain redundant information. The key challenge then is to find a less correlated image representation. Redundancy and elimination of irrelevancy are two basic elements of compression. Redundancy elimination helps to remove repetition (image/video) from the source of the signal. The reduction of irrelevance omits portions of the signal that the source of the signal, including the Human Visual System, would not perceive (HVS)[1].

Compression of images has been always significant, and because of the tremendous demand for image storage and transfer, it is now becoming necessary. Numerous and varying image compression approaches have been suggested over the last two decades. The most commonly used approaches are focused on coding based on transformations. Many image compression formats such as JPEG2000 and JPEG have been built based on transform-based strategies [2]. The commonly utilized formats for JPEG2000 and JPEG use discrete cosine transformation (DCT) and wavelet transformation to represent images compressively. Even so, for more compact image representation, JPEG and JPEG2000 do not take into consideration the spatial similarity of adjacent blocks[3]. In recent years, intra-prediction[4] schemes have played a promising role in using neighboring blocks for further compression in the compression process. Based on the information of decoded neighboring blocks, the principle of intra-prediction is to predict the unknown picture block. The average rate of coding is decreased by a successful forecast. However, there are some drawbacks to these compression techniques, since some particular types of images can not be compressed effectively. They are unable to depict the dynamic features of an image sparsely. Sparse representations have developed in recent years to solve these restrictions. Sparse representation is a very useful method for a wide number of images to be compressed. This is possible since, concerning any basis or dictionary, the pictures may be scattered or compressible. Sparse representation thus provides the opportunity for efficient compression of images. A picture is or is not compressible, depending on the dictionary, however, in sparse representation, the dictionary architecture is important[5].

The intrinsic structure identified in the natural data means that, in an effective coordinate scheme, the data allows for a sparse representation. In other phrases, to describe the patterns that are involved, and to what degree, only a few variables are needed if natural data is represented on a well-chosen base. Both data compression is based on sparsity, by which a signal is more accurately interpreted on a generalized transformation basis, such as Fourier or wavelet bases, in terms of the sparse vector of coefficients. This model has been turned upside down by recent fundamental developments in mathematics. It is already possible to achieve compressed measurements and solve for the sparsest high-dimensional signal that is compatible with the measurements rather than obtaining a high-dimensional sample and then compressing. This so-called compressed sensing is a valuable new perspective that is also relevant for complex systems in engineering, with the potential to revolutionize data acquisition and processing. In this chapter, we discuss the fundamental principles of sparsity and compression as well as the mathematical theory that enables compressed sensing, all worked out on motivating examples[6].

The major aim of compression is to remove the redundancy that is present in the original data of the image. A crucial factor for compressed sensing is also the improved sparsity of the image representation. The more sparse signal means that fewer samples are used to reproduce the signal correctly. The strong impetus for the creation of image object extraction techniques derives from the fact that almost all image details display a distinctive texture that is ignored in traditional classifications. Concerning its smoothness or its coarseness, the texture of an object may be described. One field of image processing in which texture quantification plays an important role is that of industrialized sight.

Our discussion on sparsity and compressed sensing will necessarily involve the critically important fields of optimization and statistics. Sparsity is a useful perspective to promote parsimonious models that avoid overfitting and remain interpretable because they have the minimal number of terms required to explain the data. Having a sparse representation plays a fundamental role in how well we can compress, de-noise, and restore.

2. Related Work

Significant volumes of high-dimensional data have been generated in different areas of engineering and science over the past decade. Not only the volume of information collected currently beyond the existing processing and storage capacities, but the speed at which these databases are generated is growing. Although large-scale data set acquisition, storage, and analysis face various challenges, high-dimensional sets of data have posed many research programs in fields such as visual data acquisition, bioinformatics, network analytics, biomedical imaging, and many more. Large-scale high-dimensional knowledge has shifted experimental findings to a new paradigm, defined as the fourth research paradigm [7].

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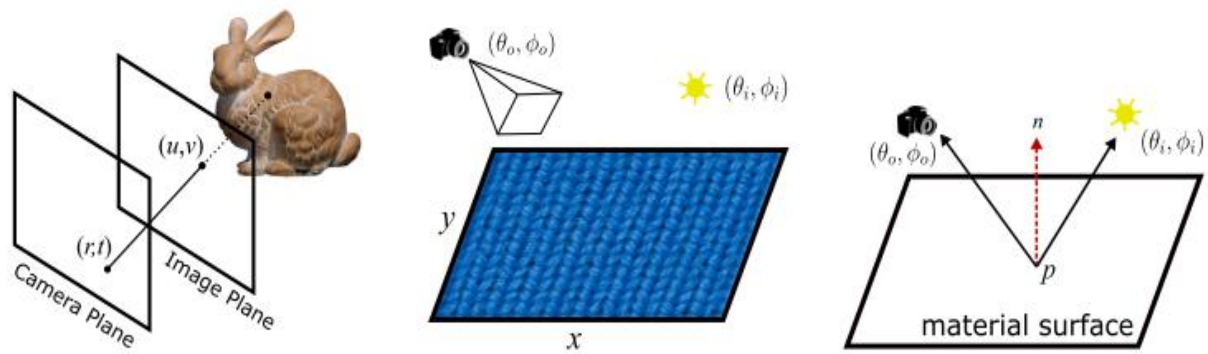
A variety of new massive-scale visual evidence such as multispectral light images and film-measured BRDFs and Spatially Changing BRDFs (SVBRDFs), multispectral images, Bidirectional Texture Functions (BTF) and Magnetic Resonance Imaging (MRI)[8] have been incorporated in the ongoing developments in imaging techniques. High dimensionality, as Figure 1, is a typical function of these data sets. 1. For example, a light field is a 5D function known as l(r, t, u,v,λ), where the spatial domain is described by (r, t), the angular domain is parametrized by (u,v), and the wavelength [9] is expressed by λ. A BRDF, depending on the way we define the dependency on wavelengths, may be parameterized as a 4D, 5D, or 6D entity. The latest developments in imaging technology have made it possible to collect these datasets in any direction with a high resolution. In addition, existing means for collecting such datasets can be expanded to handle more details, such as the acquisition of videos of multispectral light fields [9].



(a) Light field parameterized by the tuple (r,t,u,v)

(b) BTF parameterized by the tuple $(x,y,\phi_i,\theta_i,\phi_o,\theta_o)$

(c) BRDF parameterized by the tuple $(\phi_i,\theta_i,\phi_o,\theta_o)$

Figure 1: Examples of high-dimensional datasets used in computer graphics.

Novel and[10] and photo-realistic real-time rendering[11] have indeed been found for the different forms of high dimensional data mentioned above. In a large number of applications, BTFs have already been used, such as BRDF estimation[11], computational study of cast shadows, real-time rendering[12], and geometric estimation[13]. In addition, measured BRDF datasets such as[14] have made it possible for more than a decade of analysis to extract new analytical models and evaluate existing ones[15], and also to conduct effective photo-realistic representation.

With the aforementioned discussion, it is obvious that there are several steps in which high-dimensional visual data can be integrated into graphics, each containing various research paths (see Figure 2). The first aspect is to extract meaning for various applications to effectively collect multidimensional spatial data with high precision. This usually entails several sensors and cameras being used. The volume of data generated is always orders of magnitude greater than the storage device capabilities (I/O speed, capacity, etc.) for handheld measuring instruments. In comparison, encoding of large-scale broadcasting data in real-time has been either impracticable or very expensive. However, compressed data can be calculated directly, bypassing two expensive phases: that is, the handling of raw data accompanied by compression. This is the core principle behind compressed sensing, which is a comparatively recent field in applied mathematics and signal processing. [9] provides a short overview of the contributions to the study of compressed sensing in this study. In two basic fields, these innovations advance the field: theoretical and analytical [9].

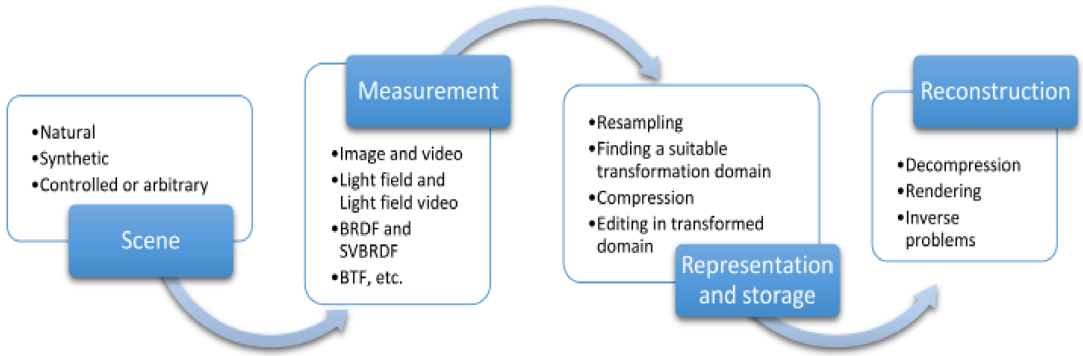


Figure 2: A pipeline for utilizing high-dimensional visual data in graphics[9].

Another significant feature of multidimensional visual information is the discovery of preliminary information representations that enable storage, retrieval, and rapid restoration. It is important to provide correct data models in order to extract acceptable basis functions for the efficient representation of the data. A significant explanation for finding data transformations is that it is too difficult to interpret, archive, or make the original domain of the information. Since the early days of computer graphics, with the ever-computing costs of these databases and the constraints of computational capacity and memory, such descriptions have become a fundamental component of several technologies. Spherical harmonics have, for example, been used for the inference and representation of radiance transition functions of BRDF[16]. For the computational study of light transport and many other applications[17],[18], the domain of Fourier has been used. In addition, wavelets have already been shown to be an effective graphics system. [19].

Sparse representations, where an image could be represented by the linear combination of a limited section of elements in the dictionary, are of special importance in many signal and image processing applications. Compression is a simple by-product of sparse representations, as a signal is modeled using a few scalars. If an adequate dictionary is provided or trained, visual evidence such as natural images and light fields identify sparse representations. The consistency of the representation is closely related to the dictionary, i.e. the sparsity and amount of error introduced by the model. In truth, the more sparse the representation, the greater the ratio of compression becomes. In this direction, this dissertation makes many contributions. In particular, we suggest algorithms that allow highly sparse multidimensional visual data representations, as well as numerous applications that use this model for the powerful compression of large-scale graphical datasets. A crucial factor for compressed sensing is also the improved sparsity of the representation. The fewer samples are needed to recreate the signal specifically, the much more fuzzy a signal is[8].

3. Methodology

3.1 Overview

The bulk of natural image signals are highly compressible, such as pictures, videos, and audio. This compressibility ensures that only a few pixels or modes are active while the signal is sorted on an acceptable basis, thus minimizing the number of values that need to be processed for a reliable representation. Said another way, a compressible signal transform basis $x \in R^n$ may be written as a sparse vector $s \in R^n$ (containing mostly zeros) in a $\Psi \in R^{n*n}$:

$$\mathbf{x} = \mathbf{\Psi s} \qquad (1)$$

In particular, if there are exactly K nonzero components, the vector s is referred to as K-sparse in Ψ. If the justification Ψ is generic, like the Fourier or wavelet basis, then it is only necessary to rebuild the original signal x in the few active terms in s, reducing the data required to store or transmit the signal.

In Fourier or wavelet bases, pictures and speech recordings are both compressible, so most coefficients are small after taking the Fourier or wavelet transformation and can be set exactly equal to zero with negligible reduction in the quality. These few active coefficients may be stored and transmitted, instead of the original high-dimensional signal. Then, to reconstruct the original signal in the ambient space (i.e., in pixel space for an image), one need only take the inverse transform. The fast Fourier transform is the enabling technology that makes it possible to efficiently reconstruct an image x from the sparse coefficients in s. This is the foundation of JPEG compression and MP3 compression for images and audio respectively.

Generic or standardized bases are the Fourier methods and wavelets, in the sense that approximately all images or audio signals in such bases are sparse. Therefore, once an image is compressed, the sparse vector s needs only to be stored or transmitted rather than the entire matrix, since the trans-forms of Fourier and wavelet are already hard-coded on most machines. Compression of signals using the SVD is also possible, resulting in a customized basis.

Although the majority of compression theory has been driven by audio, image, and video applications, there are many implications for engineering systems. The solution to a high-dimensional system of differential equations typically evolves on a low-dimensional manifold, indicating the existence of coherent structures that facilitate sparse representation. Even broadband phenomena, such as turbulence, may be instantaneously characterized by a sparse representation.

3.2 Why signals are compressible: The vastness of image space

It is important to note that the compressibility of images is related to the overwhelming dimensionality of image space. For even a simple 20×20 pixel black and white image, there are 2^{400} distinct possible images, which is larger than the whelming dimensionality of image space. The number of images is considerably more staggering for higher-resolution images with greater color depth.

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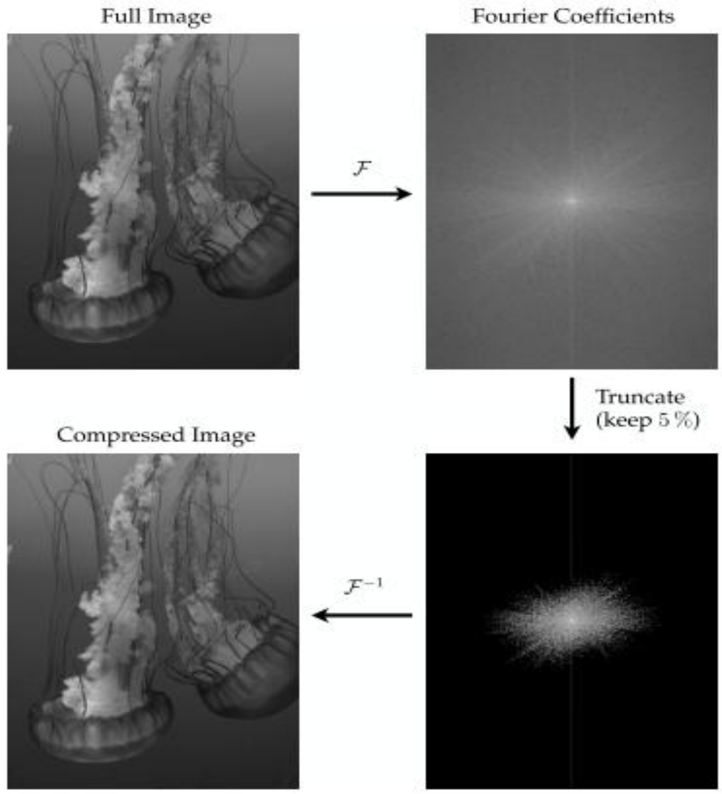

Figure 3: Illustration of compression with the fast Fourier transform.

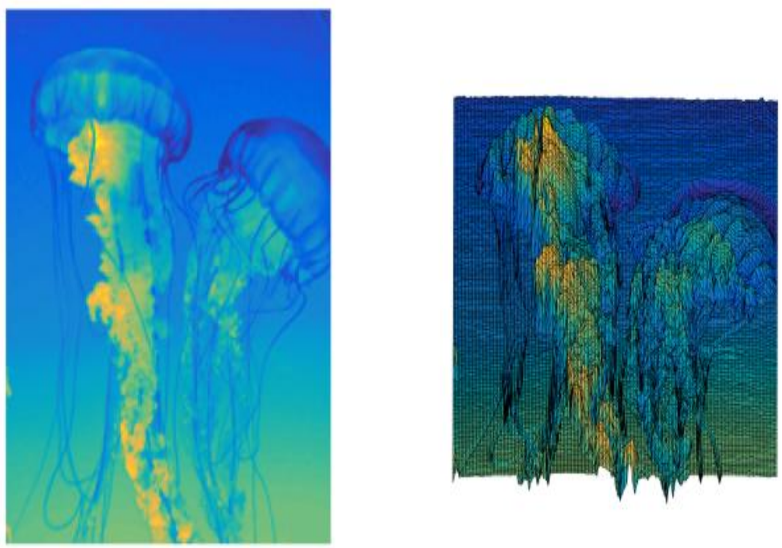

Figure 4: Compressed image (left), and viewed as a surface (right).

However vast the space of these natural images, they occupy a tiny, minuscule fraction of the total image space. The majority of the images in the image space represent random noise, resembling television static. For simplicity, consider grayscale images, and imagine drawing a random number for the gray value of each of the pixels. With exceedingly high probability, the resulting image will look like noise, with no apparent significance. You could draw these random images for an entire lifetime and never find an image of a mountain, a person, or anything physically recognizable.

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In other words, natural images are extremely rare in the vastness of image space, as illustrated in Figure 5. Because so many images are unstructured or random, most of the dimensions used to encode images are only necessary for these random images. These dimensions are redundant if all we cared about was encoding natural images. An important implication is that the images we care about (i.e., natural images) are highly compressible if we find a suitable transformed basis where the redundant dimensions are easily identified.

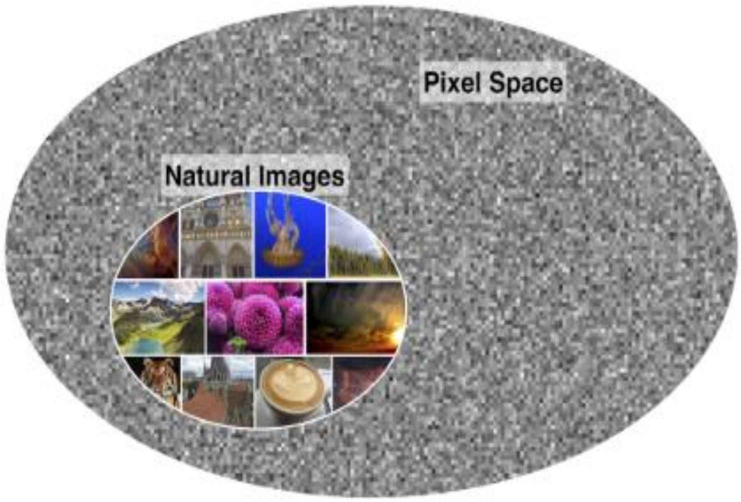

Figure 5: Illustration of the vastness of image (pixel) space, with natural images occupying a vanishingly small fraction of the space.

3.3 Compressed Sensing

Despite the tremendous progress of compression in real-world implementations, absolute, high-dimensional measurements are still available. The recent advent of compressed sensing twists the compression paradigm upside down: instead of collecting high-dimensional data only to compress and discard much of the information, you can collect remarkably few compressed measurements or random ones instead, and then figure out what is transformed by the sparse representation [20]. The idea behind compressed sensing is relatively simple to state mathematically, but until recently finding the sparsest vector consistent with measurements was a non-polynomial (NP) hard problem. The rapid adoption of compressed sensing throughout the engineering and applied sciences rests on the solid mathematical framework that provides conditions for when it is possible to reconstruct the full signal with high probability using convex algorithms.

Mathematically, compressed sensing exploits the sparsity of the image pixels on a generic basis to achieve full signal reconstruction from surprisingly few measurements. If a signal x is $\mathrm{K}-\mathrm{sparse}$ in Ψ, then rather than measuring x directly (n measurements) and then compressing, it is possible to collect dramatically fewer randomly chosen or measurements compressed and then solve for the nonzero elements of s in the transformed coordinate system. The measurements $\mathrm{y} \in \mathrm{R}^{\mathrm{p}}$, with $\mathrm{K} < \mathrm{p} \ll \mathrm{n}$ are given by:

$$\mathbf{y} = \mathbf{C}\mathbf{x} \qquad (2)$$

The measurement matrix $\mathrm{C} \in \mathrm{R}^{\mathrm{p}\times\mathrm{n}}$ performs a group of p liner's measurements on the x state. The option of matrix C measurement in compressed sensing is of paramount significance. Usually, calculations may contain random state estimates, so in that case, the C inputs are random variables distributed by Gaussian or Bernoulli. Individual entries of x can also be calculated (i.e., single pixels if x is an image) where C contains random identity matrix rows.

With knowledge of the sparse vector s it is possible to reconstruct the signal x from equation 1 The purpose of the compressed sensing is therefore to locate the sparsest vector in line mostly with parameter y:

$$\mathbf{y} = \mathbf{C\Psi s} = \mathbf{\Theta s} \qquad (3)$$

The scheme of formulas in 3 is under-determined and there are limitless implementations. The sparsest solution meets the following problem of optimization:

$$\hat{\mathbf{s}} = \underset{\mathbf{s}}{\mathbf{argmin}} \parallel \mathbf{s} \parallel_0 \ \text{subject to } \mathbf{y} = \mathbf{C\Psi s} \qquad (4)$$

where $\| \cdot \|_0$ denotes the l_0 pseudo-norm, given by the number of nonzero entries; this is also referred to as the cardinality of s. The optimization in equation 4 is non-convex, and in general, the solution could be only found with a validation for brute force that is combinatorial in n and K. In particular, all possible K-sparse vectors in R^n must be checked; if the exact level of sparsity K is unknown, the search is even broader. Because this search is combinatorial, solving equation 4 is intractable for even moderately large n and K, and the prospect of solving larger problems does not improve with Moore's law of exponentially increasing computational power.

Interestingly, the optimization can be relaxed under such constraints on the measuring matrix C in equation 4 to a convex l_1-minimization [21]:

$$\hat{\mathbf{s}} = \underset{\mathbf{s}}{\mathbf{argmin}} \parallel \mathbf{s} \parallel_1 \ \text{subject to } \mathbf{y} = \mathbf{C\Psi s} \qquad (5)$$

where $\| \cdot \|_1$ is the $l_1$norm, given by:

$$\parallel \mathbf{s} \parallel_1 = \sum_{\mathbf{k}=1}^{\mathbf{n}} |\mathbf{s}_\mathbf{k}| \qquad (6)$$

The l_1 norm is also known as the taxicab or Manhattan norm because it represents the distance a taxi would take between two points on a rectangular grid.

The overview of compressed sensing is shown schematically in Figure 6. The l_1 the minimum-norm solution is sparse.

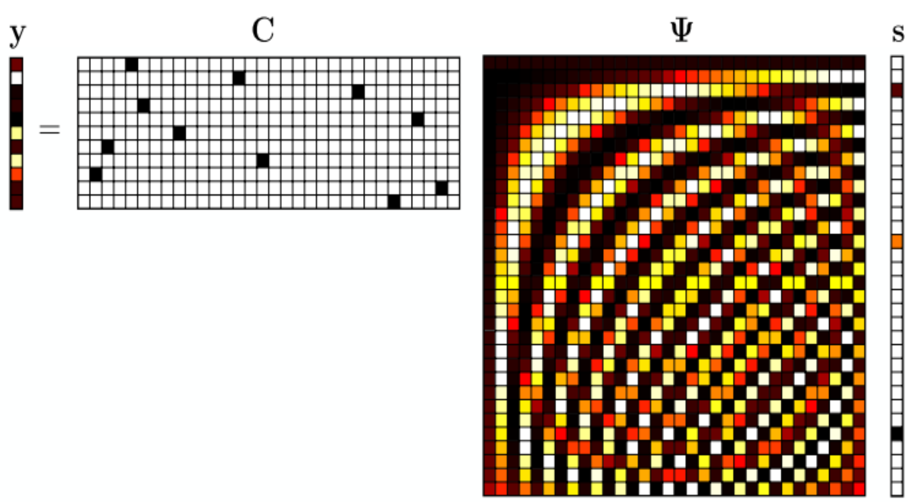

Figure 6: Schematic of measurements in the compressed sensing framework.

3.4 Working Structure

Step 1: load an image.

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Step 2: convert to grayscale and plot it.

Step 3: Apply a fast Fourier transform on the grayscale image.

Step 4: plot the coefficients on a logarithmic scale.

Step 5: arrange all the Fourier coefficients in order of magnitude and decide what percentage to keep (This sets the threshold for truncation).

Step 6: view the image as a surface To understand the role of the sparse Fourier coefficients in a compressed Image.

4. Experimental Results

Compression is relatively simple to implement on images. Starting with high resolution. After that a transformed base is used in this project, and Fast Fourier transform is used. Then by using sparsity we can truncate and throw away a vast majority of those entries, which means we keep little Fourier coefficients that contribute the most important parts of the original image then after applying inverse Fourier transform we may get an high fidelity representation of the original image. In this case, my picture is being used First, we load an image, convert it to grayscale, and plot it as in figures 7-a and 7-b respectively.

Next, we take the fast Fourier transform and plot the coefficients on a logarithmic scale as in Figure 8. To compress the image, we first arrange all of the Fourier coefficients in order of magnitude and decide what percentage to keep (in this case 5%, 10%, and 30%). This sets the threshold for truncation as in figures 9-a, 10-a, and 11-a respectively.

Finally, we plot the compressed image by taking the inverse FFT (IFFT) as in figures 9-b, 10-b, and 11-b respectively.

(a) (b)

Figure 7: The rested image (a) Original Image, (b) Converted image into grayscale.

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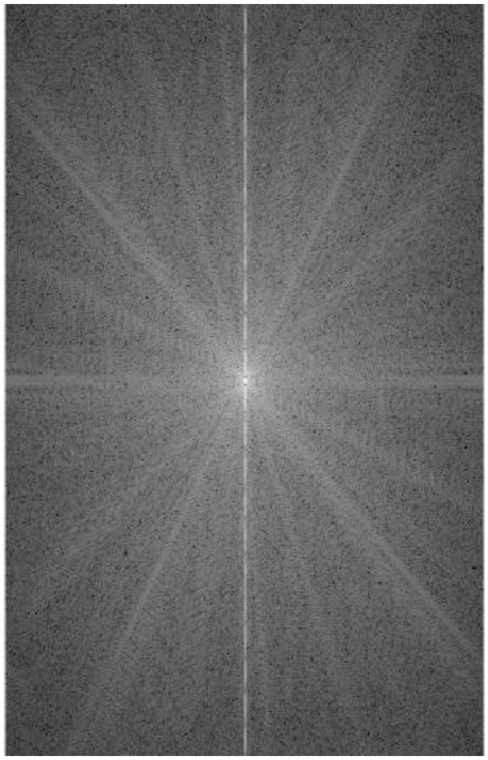
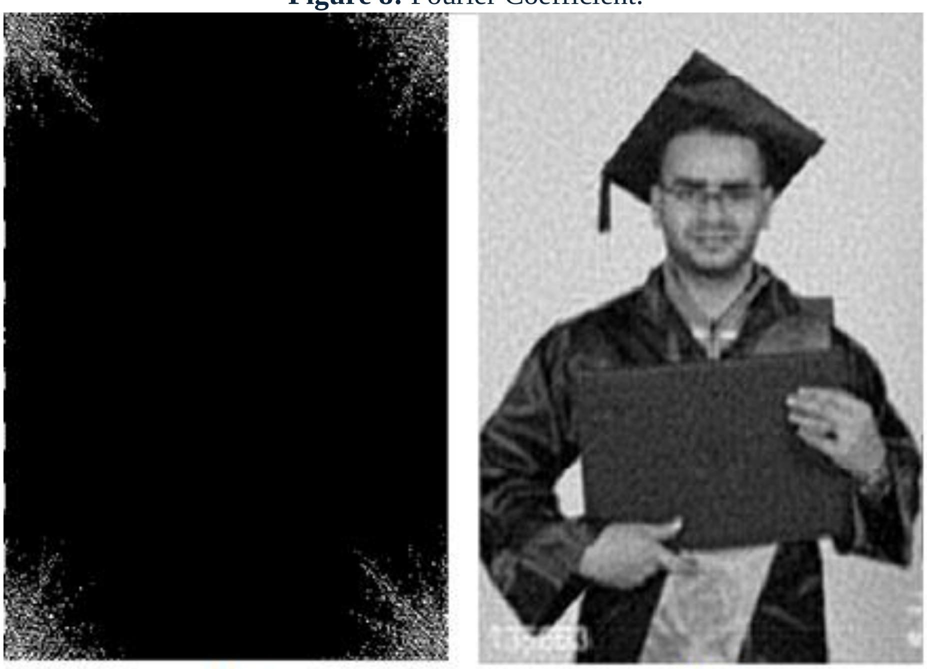


Figure 8: Fourier Coefficient.



(a) (b)

Figure 9: Keeping 5% of the pixels: (a) Sparse Signal for the Fourier Coefficient, (b) Compressed image.

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Figure 10: Keeping 10% of the pixels: (a) Sparse Signal for the Fourier Coefficient, (b) Compressed image.

Figure 11: Keeping 30% of the pixels: (a) Sparse Signal for the Fourier Coefficient, (b) Compressed image.

To understand the role of the sparse Fourier coefficients in a compressed image, it helps to view the image as a surface, where the height of a point is given by the brightness of the corresponding pixel. This is shown in Figure 12. Here we see that the surface is relatively simple, and may be represented as a sum of a few spatial Fourier modes.

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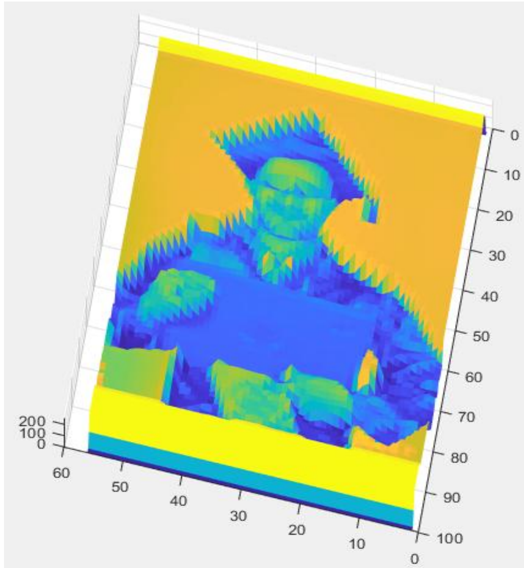

Figure 12: Compressed image as a surface.

5. Conclusion

Information and communication technology is evolving because of advanced social media networks and mobile internet technologies. Millions of images and video services are distributed every day on the Internet in different forms. Restricted by bandwidth and storage, encoding of images and videos is also essential to the media. Having a sparse representation plays a fundamental role in how well we can compress, de-noise, and restore. In this project, my image is compressed by using the sparsity method and to get almost the same efficiency and level of clearness almost 70% of the original size has almost saved.

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