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Stochastic Reliability Modelling of Three-Component Systems under Simultaneous Failure Shocks

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***Abstract*:**

In this article, Reliability analysis of a three-unit identical system is discussed. The units of system may affect by two types of failures namely, Lethal Common Cause Shock (LCCS) and Non-Lethal Common Cause Shock (NCCS) failures. Using stochastic process, the set of differential equations of the current model are derived to obtain reliability measures such as reliability of the system and Mean Time to Failure (MTTF) in the case of series and parallel. Also, the Maximum Likelihood Estimates (MLE) of the above said measures are discussed and presented in numerical illustration by using simulation. Tables display the findings and suggest that LCCS and NCCS are the most dominant causes of failures while studying the performance of the systems in reliability theory.

Keywords: **Unit system, Stochastic process, Reliability, MTTF, LCCS & NCCS Failures, M L Estimation, Simulation**

1. Introduction

Reliability is the most accepted analysis tool for solution of engineering problems. It is the parameter which is used to assess the effectiveness of the system/item and availability of the system under proper working conditions for a given period. Initially, the reliability evaluation techniques have been used in aerospace industry and military applications, there after nuclear power plants, electricity supply and continuous process plants were rapidly applied the developments of reliability techniques. Situations where the failures of some or even majority of system units could lead to partial ability or partial system down time to perform required operations are quite common in electrical/mechanical systems. These types of models are also used to describe multi-channel systems (eg. telecommunications and transportation).

While evaluating the system reliability, we need to consider the Common Cause Shock (CCS) failures which can severely degrade the reliability of devices, systems etc. These events are purely external causes which produces multiple failures. As per the reliability literature, in particular two types of CCS failures viz. Lethal common cause shock failures, which is the occurrence of simultaneous outage of all units in the system and the other is non-lethal common cause shock failures, which is the occurrence of random number of units to simultaneous outage of several units in the system. Some attempts have been made in this direction by several authors. Billinton and Allan [1] discussed the role of common cause shock failures in different frame works. Chari et al [2] derived the reliability measures of a two unit system in the presence of common cause shock failures. Dhillon [3], [4] discussed the role of common cause failures as well as human errors in system reliability aspects. Reddy [5] has developed reliability measures for two component non-identical system with common cause failures. Sagar et al [6], [8] and Awgichew et al [7] examined the reliability measurements with common cause shock failures for two unit identical system. They derived M L estimates of two unit system reliability measures such as frequency of failures in the presence of CCS failures. Sreedhar et al [9], [10] analysed two unit non identical system with CCS failures. They studied M L estimation approach for estimating reliability indices.

1.1 Model, Assumptions and Notations

In this article, as we discussed in the introduction, none of the authors studies three-unit identical systems with LCCS and NCCS failures and maximum likelihood estimation as well. We examined the reliability of the redundant system in series as well as parallel configurations. There are four different possible states for the system operation: perfect state, minor failed state, major failed state, and completely failed states. The failure rates of each unit are constant in nature, but they follow exponential distribution.

2. Related Work:

A. Notations:

s / t	–	Laplace transform/Time scale variable
$\lambda / \omega / \beta$	–	Failure rate of individual unit / LCCS / NCCS
μ_0 / μ_1	–	Repair rates
T	–	Time to failure of a unit
$p(q)$	–	The probability of simultaneous failures of units due to NCCS / LCCS
$Pi(t)$	–	Probability that the system is in state (i = 0, 1, 2, 3) at time t
$R_{LNS}(t) / R_{LNP}(t)$	–	Reliability of the system when units are in series / parallel
$\hat{R}_{LNS}(t) / \hat{R}_{LNP}(t)$	–	M L estimate of reliability function for series / parallel system
$E_{LNS}(T) / E_{LNP}(T)$	–	Mean time to failure for series / parallel system

B. Assumptions

In this paper, we consider the following assumptions:

1. The system operates effectively until one or more units are functioning.
2. The system has four states: perfect, minor partially failed, major partially failed, and completely failed.
3. The system units fail individually and also simultaneously due to lethal common cause shock failures or non-lethal common cause shock failures in Poisson manner.
4. Individual, lethal common cause shock and non-lethal common cause shock failures are independent to each other.
5. A repair man is available and ready to restore minor and major faults whether they are failed individually or simultaneously due to common cause shocks.
6. The repair times of failed units depend on the failure mode and are assumed exponentially distributed.

C. Notations:

s / t	–	Laplace transform/Time scale variable
$\lambda / \omega / \beta$	–	Failure rate of individual unit / LCCS / NCCS
μ_0 / μ_1	–	Repair rates
T	–	Time to failure of a unit
$p(q)$	–	The probability of simultaneous failures of units due to NCCS / LCCS
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$R_{LNS}(t) / R_{LNP}(t)$	–	Reliability of the system when units are in series / parallel
$\hat{R}_{LNS}(t) / \hat{R}_{LNP}(t)$	–	M L estimate of reliability function for series / parallel system
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3. Methodology

In this study, we use a probabilistic method to understand how a power system behaves when it can be in different working or failed states. We represent the system using five possible conditions—from fully functional to completely broken—and model how it moves between these conditions over time using continuous-time Markov chains (CTMC).

To do this, we create a diagram that maps out every possible change the system can go through, whether it's a breakdown or a repair. Based on how often these changes happen, we build a mathematical model (called a transition matrix) to analyze the system's long-term behavior.

We then calculate important reliability measures like how often the system is available, how long it usually works before breaking down (MTTF), and how long it takes to repair (MTTR). Finally, we test how changes in failure and repair rates affect these results, helping us pinpoint the most important factors that influence system reliability.

3.1 State Transition Diagram and Description

I. Mathematical Model

In view of the stated assumptions, we formulate state transition diagram of the model in Fig.1. The state description of the current model highlights that initially all the units are functioning perfectly and it in a state of s_0. After any one of the three units is down and others are functioning, it switches to state s_1 which is regarded as minor partially down state. If two units have failed, it will be passed to s_2 that is the major partially down state. In both cases, to restore the system we use general repair. State s_3 indicates completely down state due to failure of all the three units. The quantities that appear in Fig.1 are defined as:

$$\left.\begin{aligned} \lambda_0 &= 3(\lambda + \beta p q^2) \\ \lambda_1 &= (\beta p^3 + \omega) \\ \lambda_2 &= (\beta p^2 + \omega) \\ \lambda_c &= \lambda + \beta p \\ \mu_0 &= \mu \\ \mu_1 &= 2\mu \end{aligned}\right\} \quad (1)$$

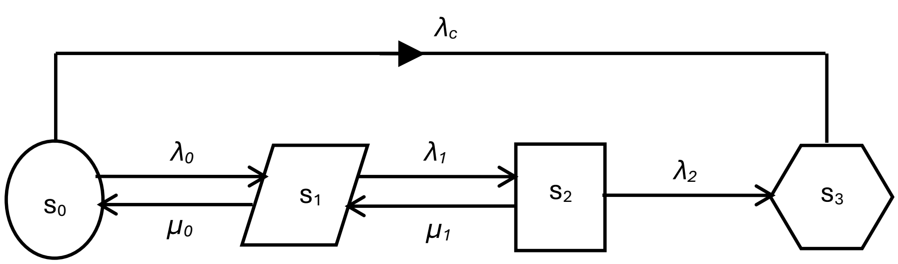

Fig.1 State transition diagram

II.

The set of differential equations associated with the current mathematical model for the above state transition diagram are:

$$P_0^{'}(t) = -(\lambda_0 + \lambda_c)P_0(t) + \mu_0 P_1(t) \tag{2}$$

$$P_1^{'}(t) = \lambda_0 P_0(t) - (\lambda_1 + \mu_0)P_1(t) + \mu_1 P_2(t) \tag{3}$$

$$P_2^{'}(t) = \lambda_1 P_1(t) - (\lambda_2 + \mu_1)P_2(t) \tag{4}$$

$$P_3^{'}(t) = \lambda_c P_0(t) + \lambda_2 P_2(t) \tag{5}$$

Initial conditions: $P_0(0) = 1$, and other state probabilities are zero at $t = 0$

Taking Laplace transformation of equations (2) to (5) and using initial conditions, we obtain

$$P_0(t) = \frac{r_1^2 + r_1 K + L}{(r_1 - r_3)(r_1 - r_2)}\exp(r_1 t) - \frac{r_2^2 + r_2 K + L}{(r_1 - r_2)(r_2 - r_3)}\exp(r_2 t) + \frac{r_3^2 + r_3 K + L}{(r_1 - r_3)(r_2 - r_3)}\exp(r_3 t) \tag{6}$$

$$P_1(t) = \frac{\lambda_0(r_1 + \lambda_2 + \mu_1)}{(r_1 - r_3)(r_1 - r_2)}\exp(r_1 t) - \frac{\lambda_0(r_2 + \lambda_2 + \mu_1)}{(r_1 - r_2)(r_2 - r_3)}\exp(r_2 t) + \frac{\lambda_0(r_3 + \lambda_2 + \mu_1)}{(r_1 - r_3)(r_2 - r_3)}\exp(r_3 t) \tag{7}$$

$$P_2(t) = \frac{\lambda_0\lambda_1}{(r_1 - r_3)(r_1 - r_2)}\exp(r_1 t) - \frac{\lambda_0\lambda_1}{(r_1 - r_2)(r_2 - r_3)}\exp(r_2 t) + \frac{\lambda_0\lambda_1}{(r_1 - r_3)(r_2 - r_3)}\exp(r_3 t) \tag{8}$$

$$P_3(t) = 1 - [P_0(t) + P_1(t) + P_2(t)] \tag{9}$$

$$\left.\begin{aligned} r_1 &= -r\sin(\alpha) - \frac{A_1}{3} \\ r_2 &= r\sin\left(\frac{\pi}{3} + \alpha\right) - \frac{A_1}{3} \\ r_3 &= r\sin\left(-\frac{\pi}{3} + \alpha\right) - \frac{A_1}{3} \end{aligned}\right\} \tag{10}$$

Here

$$\left.\begin{aligned} q &= A_3 - \frac{A_1 A_2}{3} + 2\frac{A_1^3}{27} \\ r &= \frac{2}{3}\left(A_1^2 - 3A_2\right)^{1/2} \\ \alpha &= \frac{\sin^{-1}\left(\frac{-4q}{r^3}\right)}{3} \end{aligned}\right\}$$

where

$$\left.\begin{aligned} K &= (\lambda_1 + \lambda_2 + \mu_0 + \mu_1) \\ L &= (\lambda_1\lambda_2 + \mu_0\lambda_2 + \mu_0\mu_1 \end{aligned}\right\} \tag{11}$$

$$A_1 = (\lambda_0 + \lambda_1 + \lambda_2 + \mu_0 + \mu_1 + \lambda_c)$$

$$A_2 = (\mu_0\mu_1 + \mu_0\lambda_c + \mu_1\lambda_c + \mu_1\lambda_0 + \mu_0\lambda_2 + \lambda_1\lambda_c + \lambda_2\lambda_c + \lambda_0\lambda_1 + \lambda_0\lambda_2 + \lambda_1\lambda_2)$$

$$A_3 = (\mu_0\mu_1\lambda_c + \mu_0\lambda_c\lambda_2 + \lambda_1\lambda_2\lambda_c + \lambda_0\lambda_1\lambda_2)$$

III. SOME RELIABILITY CHARACTERISTICS

In this section, we derived some performance measures when three units of the system are in series and in parallel modes.

A. Series System

In this case, all units of the system are in good working condition. The states s_1 to s_2 and s_2 to s_3 are absorbing states and hence no transition is allowed. Therefore, the reliability function is given by:

$$R_{LNS}(t) = P_0(t)$$

$$= \exp(-(4\lambda + \beta p(1+3q^2)t) \tag{12}$$

And the mean time to failure is:

$$E_{LNS}(T) = \int_0^\infty R_{LNS}(t).dt$$

$$= \frac{1}{4\lambda + \beta p(1+3q^2)} \tag{13}$$

B. Parallel System

The reliability function for parallel system is:

$$R_{LNP}(t) = P_0(t) + P_1(t) + P_2(t)$$

$$= M_1 \exp(r_1 t) - M_2 \exp(r_2 t) + M_3 \exp(r_3 t) \tag{14}$$

Where

$$M_1 = \left((r_1^2 + r_1 K + L) + \lambda_0(r_1 + \lambda_2 + \mu_1) + \lambda_0\lambda_1\right) \Big/ (r_1 - r_3)(r_1 - r_2)$$

$$M_2 = \left((r_2^2 + r_2 K + L) + \lambda_0(r_2 + \lambda_2 + \mu_1) + \lambda_0\lambda_1\right) \Big/ (r_2 - r_3)(r_1 - r_2)$$

$$M_3 = \left((r_3^2 + r_3 K + L) + \lambda_0(r_3 + \lambda_2 + \mu_1) + \lambda_0\lambda_1\right) \Big/ (r_1 - r_3)(r_2 - r_3)$$

also K, L, r_1, r_2, r_3 are defined in equations (11) and (10) and MTTF of parallel system is:

$$E_{LNP}(T) = \int_0^\infty R_{LNP}(t).dt$$

$$= \frac{-(\lambda_0\lambda_1 + \lambda_0\lambda_2 + \mu_1\lambda_0 + \lambda_1\lambda_2 + \mu_0\lambda_2 + \mu_0\mu_1)}{r_1 r_2 r_3} \qquad (15)$$

Where $\lambda_0, \lambda_1, \lambda_2, \mu_0, \mu_1$ and r_1, r_2, r_3 are defined in (1) and (10)

C. *Numerical Illustration*

For illustration purpose by fixing $\lambda = 0.01, \mu_0 = 1, \mu_1 = 1.5, p = 0.3$ and for different values of time-variable t = 0, 2, 4, 6, 8, 10, 12, 14, 16, 18 and 20 units of time, we get different values of reliability for series and parallel cases as shown in table 1.

TABLE 1: Reliability for Series and Parallel Systems

Time (t)	Series System			Parallel System		
	β=0.2, ω=0.2	β=0.3, ω=0.3	β=0.4, ω=0.5	β=0.2, ω=0.2	β=0.3, ω=0.3	β=0.4, ω=0.5
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.6863	0.5918	0.5103	0.8761	0.8293	0.7804
4	0.4710	0.3502	0.2604	0.7703	0.6908	0.6074
6	0.3233	0.2073	0.1329	0.6776	0.5756	0.4722
8	0.2219	0.1227	0.0678	0.5960	0.4796	0.3670
10	0.1523	0.0726	0.0346	0.5243	0.3997	0.2852
12	0.1045	0.0430	0.0277	0.4612	0.3330	0.2217
14	0.0717	0.0254	0.0090	0.4057	0.2775	0.1723
16	0.0492	0.0150	0.0046	0.3569	0.2313	0.1339
18	0.0338	0.0089	0.0023	0.3139	0.1927	0.1041
20	0.0232	0.0053	0.0012	0.2761	0.1605	0.0809

TABLE 2: MTTF FOR SERIES AND PARALLEL SYSTEMS

$\mu = 1, p = 0.2$

λ	Series System (β, ω)			Parallel System (β, ω)		
	(0.1, 0.1)	(0.2, 0.2)	(0.3, 0.4)	(0.1, 0.1)	(0.2, 0.2)	(0.3, 0.4)
0.01	10.163	6.378	4.647	35.702	21.605	14.702
0.02	7.225	5.081	3.918	27.459	18.452	13.131
0.03	5.605	4.223	3.387	22.542	16.215	11.925
0.04	4.579	3.613	2.983	19.275	14.544	10.970
0.05	3.869	3.157	2.665	16.947	13.250	10.195
0.06	3.351	2.803	2.408	15.205	12.218	9.553
0.07	2.955	2.520	2.197	13.851	11.375	9.014
0.08	2.643	2.289	2.019	12.769	10.674	8.553
0.09	2.390	2.097	1.868	11.885	10.082	8.155
0.10	2.182	1.935	1.739	11.149	9.575	7.809

IV. ESTIMATION AND SIMULATION

A. *Estimation*

In this, we have attempted Maximum likelihood estimation to estimate the system reliability and MTTF of the present model. However, the system is under the influence of NCCS and LCCS failures in addition to individual failures.

Let the samples $x_1, x_2, ..., x_n$; $y_1, y_2, ..., y_n$ and $w_1, w_2, ..., w_n$ with size 'n' representing times between individual, NCCS and LCCS failures which will obey exponential law.

Let the samples $z_{11}, z_{12}, \ldots\ldots\ldots, z_{1n}$; $z_{21}, z_{22}, \ldots\ldots\ldots, z_{2n}$ with size '*n*' number of times between repairs of the units with exponential population law.

$\hat{\hat{x}}, \hat{\hat{y}}, \hat{\hat{w}}, \hat{\hat{z}}_1, \hat{\hat{z}}_2$ are the maximum likelihood estimates of $\lambda, \beta, \omega, \mu_0, \mu_1$ respectively.

Where, $\hat{\hat{x}} = \frac{1}{\bar{x}}; \hat{\hat{y}} = \frac{1}{\bar{y}}; \hat{\hat{w}} = \frac{1}{\bar{w}}; \hat{\hat{z}}_1 = \frac{1}{\bar{z}_1}; \hat{\hat{z}}_2 = \frac{1}{\bar{z}_2}; \bar{x} = \frac{\sum x_i}{n}; \bar{y} = \frac{\sum y_i}{n}\ \bar{w} = \frac{\sum w_i}{n}; \bar{z}_1 = \frac{\sum z_{1i}}{n}; \bar{z}_2 = \frac{\sum z_{2i}}{n}$

B. Simulation

We compute M L estimates such as $\hat{R}_{LNS}(t), \hat{R}_{LNP}(t)$ of the present model by using Monte-Carlo simulation. For a range of specified values of the rates of $\lambda, \beta, \omega, \mu_0, \mu_1$ and for the sample size n=5(5)15 were simulated in each case with N=20000(30000)100000 in order to evolve mean square error (MSE) in each case by using C++ (software).

4. Result Discussion

This paper analyzes the reliability measures of a three unit system in series and parallel under the lethal and non-lethal common cause shock failures. A study of the model with the support of maximum likelihood estimation were presented and established empirically. The importance of LCCS and NCCS failures in these types of models were discussed through numerical illustration and simulation validity in this article. The following decision can be made based on the analysis carried out in this paper.

Table 1 show evidence for the reliability of the system at various time values. The reliability is decreasing in both series and parallel cases when LCCS and NCCS failure rates are increasing. Table II include the variation in the MTTF corresponding to different failure rates in series and parallel system. It is observed that MTTF decreases as the failure rate increases and also there is a great improvement from series to parallel system. Table III and Table IV show the simulation study in order to establish the validity of the proposed maximum likelihood estimates. It is observed that the point estimates become more accurate when the sample size is large and mean square error decreases with increasing the sample size.

The model discussed in this article was found to be great importance in proper maintenance analysis, and performance evaluation of the system.

4.1 Tables

TABLE 3: RELIABILITY ESTIMATION FOR SERIES SYSTEM

$\lambda = 0.1,\ \beta = 0.2,\ \omega = 0.3,\ p = 0.3,\ t = 1$
SAMPLE SIZE (n = 5)

N	$R_{LNS}(t)$	$\hat{R}_{LNS}(t)$	M S E
20000	0.577989	0.492346	0.025525
50000	0.577989	0.491844	0.025762
80000	0.577989	0.492411	0.025832

SAMPLE SIZE (n = 10)

N	$R_{LNS}(t)$	$\hat{R}_{LNS}(t)$	M S E
20000	0.577989	0.522508	0.011552
50000	0.577989	0.523343	0.011446
80000	0.577989	0.523294	0.011507

SAMPLE SIZE (n = 15)

N	$R_{LNS}(t)$	$\hat{R}_{LNS}(t)$	M S E
20000	0.577989	0.532862	0.007407
50000	0.577989	0.533398	0.007394
80000	0.577989	0.533402	0.007395

TABLE 4: RELIABILITY ESTIMATION FOR PARALLEL SYSTEM

$\lambda = 0.1$, $\beta = 0.2$, $\omega = 0.3$, $\mu_0 = 1$, $\mu_1 = 1.5$, $p = 0.3$, $t = 1$
SAMPLE SIZE (n = 5)

N	$R_{LNP}(t)$	$\hat{R}_{LNP}(t)$	M S E
20000	0.868255	0.830932	0.003984
50000	0.868255	0.831099	0.004021
80000	0.868255	0.831404	0.003957

SAMPLE SIZE (n = 10)

N	$R_{LNP}(t)$	$\hat{R}_{LNP}(t)$	M S E
20000	0.868255	0.846471	0.001455
50000	0.868255	0.846431	0.001455
80000	0.868255	0.846591	0.001446

SIZE (n = 15)

N	$R_{LNP}(t)$	$\hat{R}_{LNP}(t)$	M S E
20000	0.868255	0.851125	0.000879
50000	0.868255	0.850872	0.000895
80000	0.868255	0.851039	0.000893

5. Conclusion

In this study, we developed a practical and detailed model to understand how a power system performs over time, especially when it can operate in more than just “working” or “failed” states. We broke the system down into five possible conditions—from fully working to completely out of service—and used a method called continuous-time Markov chains to track how it moves between these states.

By creating a transition map and analyzing how often the system shifts from one state to another, we were able to calculate important performance measures like how often the system is available, how long it works before failing, and how long it takes to fix. Our analysis showed that these changes, especially how quickly the system fails or gets repaired, play a big role in overall reliability.

This approach not only gives a clearer picture of how power systems behave in real life but also helps engineers design systems that are more dependable and easier to maintain.

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